TRANSACTIONS ON SCIENCE AND TECHNOLOGY

Viscous Dissipation Effect on the Mixed Convection Boundary Layer Flow towards Solid Sphere

Muhammad Khairul Anuar Mohamed[1], Norhafizah Mohd Sarif[1], Nor Aida Zuraimi Md Noar[1], Mohd Zuki Salleh[1,*] & Anuar Ishak[2]

[1]Applied & Industrial Mathematics Research Group, Faculty of Industrial Science & Technology, Universiti Malaysia Pahang, 26300 Kuantan, Pahang, MALAYSIA.

[2]School of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, MALAYSIA.

*Corresponding author: zukikuj@yahoo.com; Tel: +609-5492203; Fax: +609-5492766.

Received: *20 March 2016*
Revised: *26 April 2016*
Accepted: *29 April 2016*
In press: *2 May 2016*
Online: *30 June 2016*

Keywords:
Mixed convection; Solid sphere; Viscous dissipation

Abstract

In this study, the steady mixed convection boundary layer flow and heat transfer towards solid sphere with viscous dissipation effect is considered. The non linear parabolic partial differential equations are transformed before being solved numerically by Keller-box method. The effects of different Prandtl number values, the parameter of mixed convection and Eckert number are elaborated. The presence of viscous dissipation effect reduced the Nusselt number which physically promoted a conduction heat transfer process along the sphere surface. Further, the Prandtl number gives more significant impact on reduction of thermal boundary layer compared to mixed convection parameter.

Introduction

Convective flow on sphere surface becomes an important topic considered. Due to its contributions in engineering and industrial applications such as the spherical storage tanks, the vaporization and condensation of fuel droplets, packed beds in a chemical reactor or distillation process and in many electronic component that nearly spherical. The efficiency of heat transfer process (convective) depends on various factors including the design, the flow, conductivity, viscosity and the characteristic of the fluid used.

The experimental study on mixed convection boundary layer regarding the sphere surface are first done by Yuge (1960). The analytical and experimental study then has been proposed later by Hieber and Gebhart (1969) with considering the Reynolds and Grashof numbers in tiny scale. Next, Chen and Mucoglu (1977) and Mucoglu and Chen (1978) solved this topic by using approximation technique. The Huge Reynolds and Grashof values are considered with two boundary conditions which is constant wall temperature and constant heat flux. Later, Nazar *et al.* (2002) updated the works of Chen and Mucoglu (1977). The numerical solution from the stagnation point $\left(x=0°\right)$ to $x=120°$ is obtained.

The objectives of the present study is to update the work of Nazar *et al.* (2002), with considering the effects of viscous dissipation on a solid sphere. The viscous dissipation is easily to understand as the phenomena where the induced kinetic energy is converted into thermal energy. It is important to be study in order to understand the behavior of temperature distributions when the internal friction is not negligible (Gebhart, 1962). Recent researchers considered this effect are Haque *et al.* (2015) and Hashim *et al.* (2015) who studied the free convection on a sphere with temperature dependent thermal conductivity and forced convection on a stagnation flow over a stretching surface with slip effect, respectively.

Mathematical formulation

Consider the solid sphere with radius a heated to a constant temperature T_w embedded in an incompressible viscous fluid with ambient temperature T_∞ as shown in Figure 1. The $\bar{x}$ and $\bar{y}$ are orthogonal coordinates where $\bar{x}$ is measured along the sphere surface while $\bar{y}$ normal to the sphere surface. The dimensional boundary layer equations (Nazar *et al*., 2002) are:

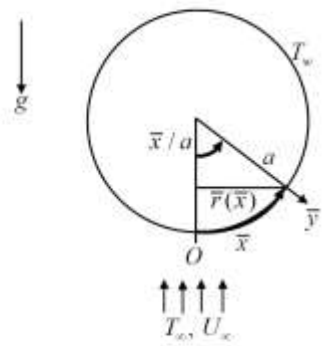

Figure 1. Physical model of the coordinate system

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u})+\frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v})=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}}+\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T-T_\infty)\sin\frac{\bar{x}}{a}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2, \tag{3}$$

with boundary conditions

$$\begin{gathered}\bar{u}(\bar{x},0)=\bar{v}(\bar{x},0)=0,\ T(\bar{x},0)=T_w,\\ \bar{u}(\bar{x},\infty)\to\bar{u}_e,\ T(\bar{x},\infty)\to T_\infty,\end{gathered} \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are the components of velocity in the directions of $\bar{x}$ and $\bar{y}$, respectively. $\bar{u}_e(x)=U_\infty\sin\left(\frac{\bar{x}}{a}\right)$ and $\bar{r}(\bar{x})=a\sin\left(\frac{\bar{x}}{a}\right)$ is the local free stream velocity and the radial distance from the symmetrical axis to the surface, respectively. μ is the dynamic viscosity, g is the gravity acceleration, ν is the kinematic viscosity, α is the thermal diffusivity, β is the thermal expansion,

T is the local temperature, ρ is the fluid density and C_p is the specific heat capacity at a constant pressure. Next, the governing non-dimensional variables are introduced:

$$x=\frac{\bar{x}}{a},\quad y=\mathrm{Re}_x^{1/2}\frac{\bar{y}}{a},\quad u=\frac{\bar{u}}{U_\infty},\quad v=\mathrm{Re}_x^{1/2}\frac{\bar{v}}{U_\infty},$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad r=\frac{\bar{r}}{a},\quad u_e(x)=\frac{\bar{u}_e(x)}{U_\infty}. \tag{5}$$

where θ is the rescaled dimensionless temperature of the fluid and $\mathrm{Re}_x=\dfrac{U_\infty a}{\nu}$ is Reynolds number.

Substitute (1)-(4) with (5), the non-dimensional governing equations is obtained:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\partial^2 u}{\partial y^2}+\lambda\theta\sin x, \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{8}$$

where $\lambda=\dfrac{Gr_x}{\mathrm{Re}_x^2}$ is mixed convection parameter, $Gr_x=\dfrac{g\beta(T_w-T_\infty)a^3}{\nu^2}$ is the Grashof number, $\mathrm{Pr}=\dfrac{\nu}{\alpha}$ is the Prandtl number and $Ec=\dfrac{U_\infty{}^2}{C_p\left(T_w-T_\infty\right)}$ is an Eckert number. Notice that $\mu=\nu\rho$. Boundary conditions (4) become

$$u(x,0)=0,\ v(x,0)=0,\ \theta(x,0)=1,$$
$$u(x,\infty)\to u_e,\ \theta(x,\infty)\to 0 \tag{9}$$

In order to solve (6) to (8), the following function is introduced:

$$\psi=xr(x)f(x,y),\quad \theta=\theta(x,y), \tag{10}$$

where $u=\dfrac{1}{r}\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{1}{r}\dfrac{\partial\psi}{\partial x}$ which identically satisfies (6). Substitute (10) into (6)-(8), the following transformed partial differential equations are obtained:

$$\frac{\partial^3 f}{\partial y^3}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\left(\frac{9}{4}\cos x+\lambda\theta\right)\frac{\sin x}{x}=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{11}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}-xEc\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right), \tag{12}$$

with boundary conditions

$$f(x,0)=0,\quad \frac{\partial f}{\partial y}(x,0)=0,\quad \theta(x,0)=1,$$
$$\frac{\partial f}{\partial y}(x,\infty)\to\frac{3}{2}\frac{\sin x}{x},\quad \theta(x,\infty)\to 0. \tag{13}$$

The physical quantities considers are the skin friction coefficient C_f and the local Nusselt number Nu_x which are given by

$$C_f = \frac{\tau_w}{\rho U_\infty^{\ 2}}, \qquad Nu_x = \frac{a q_w}{k(T_w - T_\infty)}, \tag{14}$$

The surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \qquad q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \tag{15}$$

with k being the thermal conductivity, respectively. Using (5) and (11), substitute into (14) give:

$$C_f \operatorname{Re}_x^{1/2} = \left(x\frac{\partial^2 f}{\partial y^2}\right)_{\bar{y}=0} \text{ and } Nu_x \operatorname{Re}_x^{-1/2} = -\left(\frac{\partial \theta}{\partial y}\right)_{\bar{y}=0} \tag{16}$$

Numerical method

Equations (11)-(13) are solved by using the finite difference scheme known as the Keller-box method. Keller-box method involves four main steps which begin with the reduction of (11) - (13) to a first order system. Then the difference equations using central differences is written before the resulting algebraic equations is linearized by Newton's method. Lastly, the linear systems of equations in a matrix-vector form are solved by the block triadiagonal elimination technique. The algorithm is encoded in MATLAB software for numerical computation. In this problem, the wide range of boundary layer thickness y_∞ from 6 to 20 are used in order to obtained the asymptotically results.

Result and discussion

The numerical computation by Keller-box method consider three parameters which known as the Prandtl number $\Pr$, mixed convection parameter λ and Eckert number Ec. The solution is obtained until $x = 120°$ due to the probability of boundary layer separation occur after this point. Table 1 shows the comparison values with previous published results with a very good agreement. Authors believe that Keller-box method is suitable to solve this convective boundary layer problem.

Table 2 presents the value of $Nu_x \operatorname{Re}_x^{-1/2}$ against x with various values of λ. The negatives value of λ $(\lambda < 0)$ indicates the opposing buoyant flow while $\lambda > 0$ for assisting buoyant flow. From table, it is observed that $Nu_x \operatorname{Re}_x^{-1/2}$ is a decreasing function against x. The convection heat transfer is at the maximum capability at the stagnation region $(x = 0)$. This capability decreases across the sphere surface which allowed the enhancement in conduction heat transfer capabilities. Further, it is concluded that the increased of λ delayed the separation.

Next, Figure 2 presents the variation of reduced skin friction coefficient $C_f \operatorname{Re}_x^{1/2}$ with different values of Ec. At the early stage, $C_f \operatorname{Re}_x^{1/2}$ is unique for all of Ec value. $C_f \operatorname{Re}_x^{1/2}$ gives small increment from the middle of sphere until end as Ec increases. Figure 3 illustrates the variations of reduced Nusselt number $Nu_x \operatorname{Re}_x^{-1/2}$ with different values of Ec. It is seen that $Nu_x \operatorname{Re}_x^{-1/2}$ is a decreasing function against x. Further, at the middle of sphere, the increase of Ec results to the decrease in $Nu_x \operatorname{Re}_x^{-1/2}$.

Table 1. Comparison values of $Nu_x \operatorname{Re}_x^{-1/2}$ with previous published results for different values of x and λ when $\Pr = 0.7$, $Ec = 0$.

x/λ	Nazar et al. (2002)			Present		
	-1.0	0	1.0	-1.0	0	1.0
0°	0.7870	0.8162	0.8463	0.7858	0.8150	0.8406
10°	0.7818	0.8112	0.8371	0.7809	0.8103	0.8362
20°	0.7669	0.7974	0.8239	0.7615	0.7967	0.8232
30°	0.7422	0.7746	0.8024	0.7419	0.7741	0.8018
40°	0.7076	0.7429	0.7725	0.7074	0.7425	0.7721
50°	0.6624	0.7022	0.7345	0.6624	0.7032	0.7354
120°	0.6055	0.6525	0.6887	0.6072	0.6521	0.6897
70°	0.5334	0.5934	0.6352	0.5356	0.5946	0.6346
80°	0.4342	0.5236	0.5742	0.4375	0.5249	0.5753
90°		0.4398	0.5060		0.4413	0.5071
100°		0.3263	0.4304		0.3284	0.4313
110°			0.3458			0.3466
120°			0.2442			0.2485

Table 2. Values of $Nu_x \operatorname{Re}_x^{-1/2}$ against x for various values of λ when $\Pr = 7$, $Ec = 0.1$.

x/λ	-1.0	-0.5	0	0.5	1.0	2.0
0°	1.8537	1.8742	1.8940	1.9130	1.9313	1.9662
10°	1.8209	1.8408	1.8598	1.8780	1.8955	1.9287
20°	1.7289	1.7468	1.7637	1.7796	1.7947	1.8228
30°	1.5900	1.6047	1.6182	1.6305	1.6417	1.6615
40°	1.4218	1.4326	1.4417	1.4492	1.4553	1.4640
50°	1.2431	1.2500	1.2543	1.2565	1.2568	1.2524
80°	1.0699	1.0742	1.0745	1.0716	1.0659	1.0478
70°	0.9146	0.9201	0.9189	0.9127	0.9023	0.8718
80°	0.7593	0.7775	0.7813	0.7758	0.7634	0.7238
90°	0.5295	0.6266	0.6553	0.6604	0.6522	0.6107
100°			0.4935	0.5453	0.5555	0.5257
110°				0.3483	0.4451	0.4550
120°					0.2051	0.3835

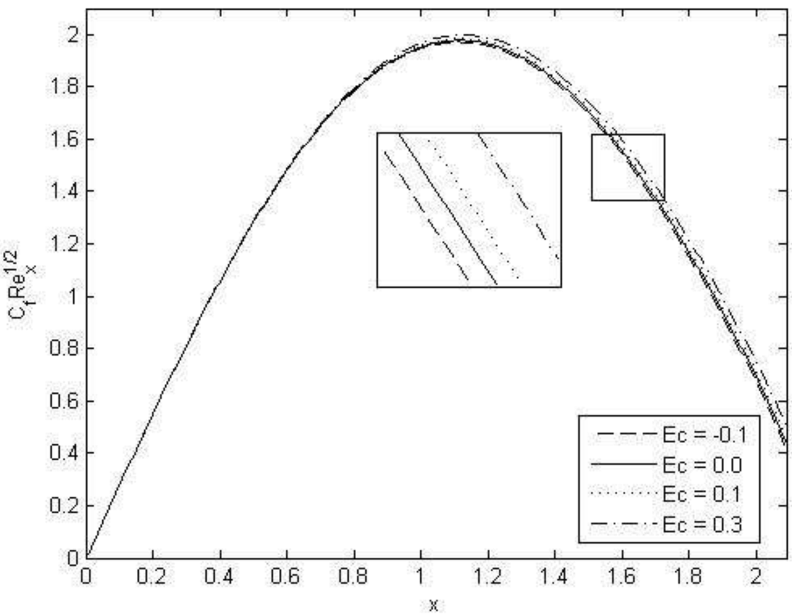

Figure 2. Variation of $C_f \mathrm{Re}_x^{1/2}$ against x for different values of Ec.

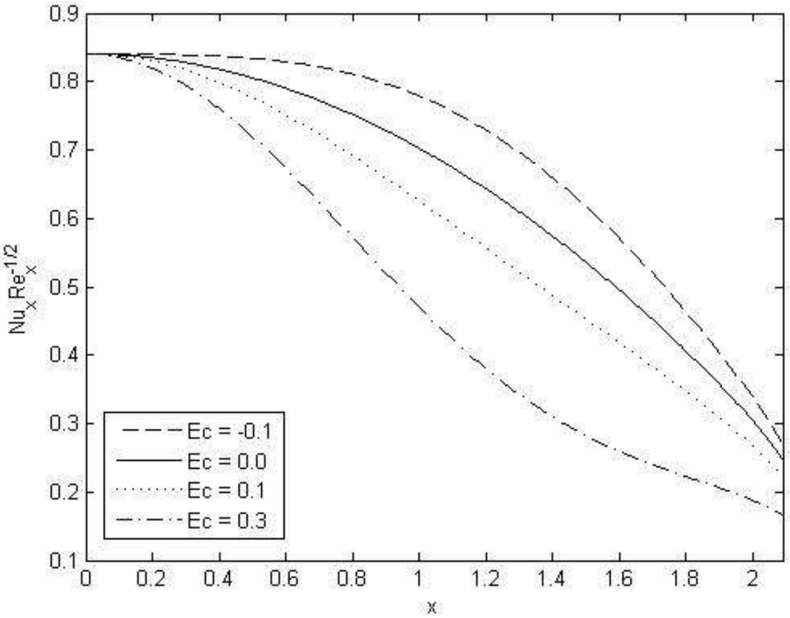

Figure 3. Variation of $Nu_x \mathrm{Re}_x^{-1/2}$ against x for different values of Ec.

Figures 4 and 5 displayed the temperature and velocity profiles for different values of Pr and λ, respectively. From Figure 4, it is suggested that the increase of Pr or λ have reduce the thermal boundary layer thickness. It is due to decrease in thermal diffusivity which reduced the energy ability and the thermal boundary layer thickness. In Figure 5, the increase of Pr results to the decrease in boundary layer thickness with the presence of λ. Meanwhile, the increase in λ enhanced the velocity gradient which physically enhanced $C_f \mathrm{Re}_x^{1/2}$.

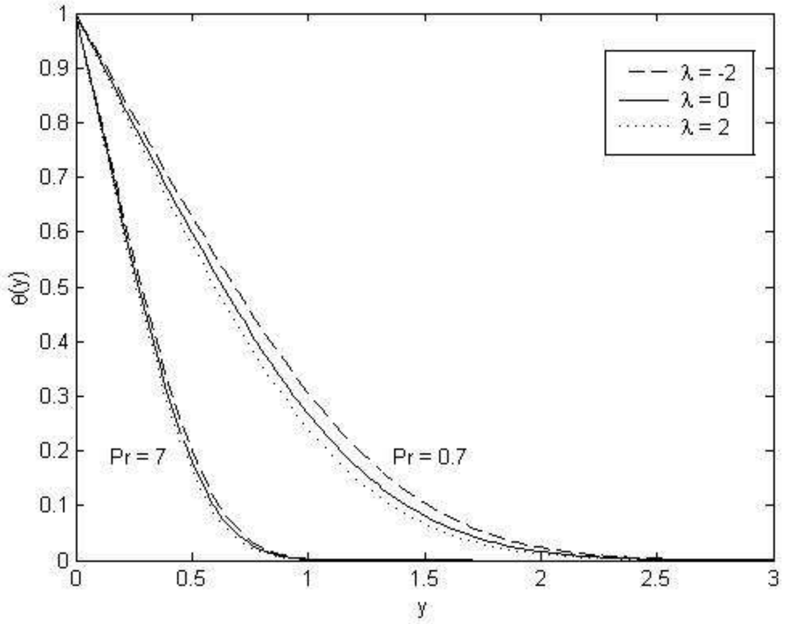

Figure 4. Temperature profiles $\theta(y)$ against y for different values of Pr and λ.

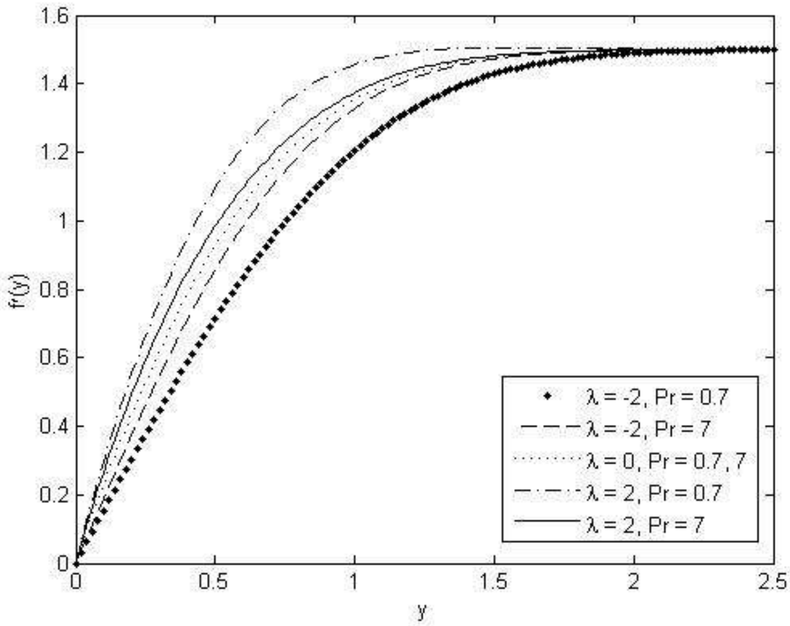

Figure 5. Velocity profiles $f'(y)$ against y for different values of Pr and λ.

Lastly, Figure 6 presents the variation of reduced skin friction coefficient $C_f \mathrm{Re}_x^{1/2}$ with different values of λ. From the figure, it is found that the increase of λ results to the increase of $C_f \mathrm{Re}_x^{1/2}$ and delayed the separation. Across the sphere surface, $C_f \mathrm{Re}_x^{1/2}$ is increase before turning decreases approaching $C_f \mathrm{Re}_x^{1/2} = 0$.

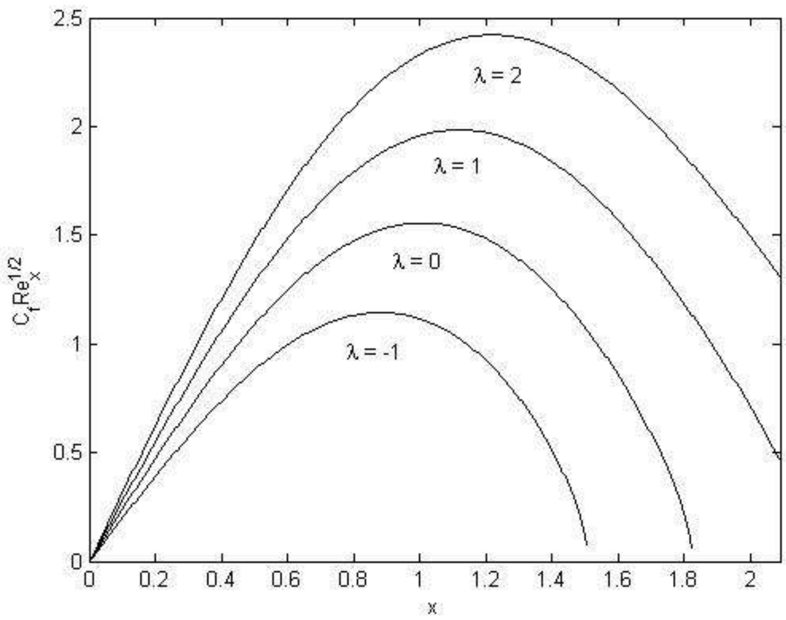

Figure 6: Variation of $C_f \,\mathrm{Re}_x^{1/2}$ against x for different values of λ.

Conclusion

As a conclusion, it is found that the presence of viscous dissipation gives small increment in skin friction coefficient from the middle until end of sphere. Meanwhile, the Nusselt number decreases with the presence of this parameter which physically denoted the reducing in convective heat transfer capabilities.

Furthermore, the increase in Prandtl number and mixed convection parameter has reduced the thermal boundary layer thicknesses. Note that, the velocity gradient which physically refers to skin friction coefficient maybe enhanced with the increase of mixed convection parameter.

Acknowledgements

Authors thank University Malaysia Pahang for the financial supports with code number RDU140111 (FRGS) and RDU150101 (FRGS).

References

[1] Chen, T. S. & Mucoglu, A. (1977). Analysis of Mixed, Forced and Free Convection about a Sphere. *International Journal of Heat and Mass Transfer*, **20**, 867-875.

[2] Gebhart, B. (1962). Effects of viscous dissipation in natural convection. *Journal of Fluid Mechanics*, **14**(2), 225-232.

[3] Haque, M. R., Alam, M. M., Ali, M. M. & Karim, R. (2015). Effects of Viscous Dissipation on Natural Convection Flow Over a Sphere with Temperature Dependent Thermal Conductivity in Presence of Heat Generation. *Procedia Engineering*, **105**(0), 215-224.

[4] Hashim, H., Mohamed, M. K. A., Hussanan, A., Ishak, N., Sarif, N. M. & Salleh, M. Z. (2015). The effects of slip conditions and viscous dissipation on the stagnation point flow over a stretching sheet. *AIP Conference Proceedings*, **1691,** 040007.

[5] Hieber, C. A. & Gebhart, B. (1969). Mixed Convection from a Sphere at Small Reynolds and Grashof Numbers. *Journal of Fluid Mechanics*, **38**, 137-159.

[6] Mucoglu, A. & Chen, T. S. (1978). Mixed convection about a sphere with uniform surface heat flux. *Journal of Heat Transfer*, **100**, 542-544.

[7] Nazar, R., Amin, N. & Pop, I. (2002). On the Mixed Convection Boundary Layer Flow about an Isothermal Sphere. *The Arabian Journal for Science and Engineering*, **27**(2C), 117-135.

[8] Yuge, T. (1960). Experiments on Heat Transfer from Spheres Including Combined Natural and Forced Convection. *Journal of Heat Transfer*, **82**, 214-220.